Functional Quantum Theory of the Nonlinear Spinor Field as a Lepton-Hadron Model with Quark-Confinement

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The non-canonical Heisenberg dipole regularization of a nonlinear spinor field is interpreted as a system of lepton quark pairs in the one-particle sector. In the framework of functional quantum theory a method is given to derive an independent lepton-quark dynamics with quark confinement from the fundamental spinor field.

Functional quantum theory is a new formulation of quantum theory and a new field theoretic calculational procedure which allows the treatment of quantized fields with positive metric as well as with indefinite metric beyond perturbation theory. It was developed by Stumpf and coworkers, cf. Stumpf [1]. In particular, it was devoted to the evaluation of Heisenberg's quantized nonlinear spinor equation with dipole ghost regularization, cf. Heisenberg [2]. The basic idea of this approach is to regularize the non-renormalizable nonlinear spinor field by a combination of a real physical particle, a monopole ghost particle and a dipole ghost particle in the one-particle sector of the corresponding state space. This leads to an indefinite metric in this space. In order to achieve a correct physical and probabilistic interpretation of this regularization procedure, the ghost particles must have the following properties:

i) To any physical particle state with positive norm in the one-particle sector belongs one monopole ghost state and one dipole ghost state which have to be combined in such a way that the propagator of the spinor field contains no singular distributions.

ii) The monopole ghost states must have vanishing norm and have to be included in the physical state space, while the dipole ghosts have to form the unphysical part of the state space.

The latter condition leads immediately to unitarization. The weakest assumption with respect to unitarization is that the dipole ghosts are not allowed to form ingoing or outgoing states in scattering processes. Heisenberg [2] demonstrated for the Lee model that under this assumption unitarization is possible. To perform this kind of unitarization for the relativistic nonlinear spinor field, functional quantum theory is needed. It should be noted that under this assumption no local physical and probabilistic interpretation of the quantized spinor field is possible. Rather this interpretation is confined to the asymptotics, i.e. to the S-matrix. Furthermore, it is tacitly assumed that the many-particle sectors of the state space which contain besides scattering states also bound states, do not show any irregular behavior, i.e. that from these sectors only the free many-particle dipole ghost states have to be excluded while all other states must be physical ones. This, of course, has to be proven by direct construction as for bound states no general statements can be made. For this construction functional quantum theory is needed.

With respect to the identification of the physical particles resulting from the quantized spinor field, the physical particles of the one-particle state sector should be identified with the pointlike particles occurring in nature, while all other particles are considered to be bound states of the spinor field. In the original version of Heisenberg [2] the physical one-particle states were assumed to represent nucleons, i.e . the theory was based on the idea that the hadrons are the fundamental pointlike particles, while, without detailed discussion, the leptons were sometimes identified with the dipole ghosts, cf. Heisenberg [2], Dürr [3]. The recent developments of elementary particle physics, however, indicate that just the opposite role of leptons and hadrons might be successful for an explanation of the elementary particle spectrum, i.e. that the leptons should be identified with the fundamental particle states of the one-particle sector with positive norm, while quasi-hadrons should be considered as ghost particle states with indefinite metric.

The present experimental identification rests on the following scheme of leptons and quarks, Harari [4], Zichichi [5]

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L \quad \begin{pmatrix}u\\ d\end{pmatrix}_L \begin{pmatrix}c\\ s\end{pmatrix}_L \begin{pmatrix}t\\ b\end{pmatrix}_L$$

and corresponding right-handed singlets.

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In order to associate the one-particle sector of the nonlinear spinor theory to this scheme, one has to observe that

α) the regularization of the propagator can also be achieved if several couples of monopole-dipole ghost particles are associated to one real physical particle, (color)

β) integer charges can be assigned to the quarks, if these particles were not related to SU(3), cf. Abers and Lee [6].

Due to α) and β) it is then reasonable to associate one physical particle state in the one-particle sector of the nonlinear spinor theory to any lepton state and to identify the one or several corresponding dipole ghosts with the corresponding quarks. In the framework of the nonlinear spinor theory the quarks are not assumed to belong to an SU(3) representation, rather their transformation properties should be governed by the electro-weak interactions, while the approximate SU(3) symmetry of the hadron spectrum should result from dynamical bound state formation alone, i.e. from three-quark systems. The masses and transformation properties of the monopole ghosts are fixed, as for a successful regularization these quantities are definitely referred to the dipole ghost states. A physical interpretation of the monopole ghosts is not necessary as they have a vanishing norm in the physical sector.

The choice of the quarks as dipole ghosts immediately solves the problem of confinement, as due to ii) no ingoing or outgoing quarks may occur. It is assumed that the combination of quarks, leptons and corresponding bosons in a common coupling gauge field theory should suffice to explain the elementary particle spectrum and reactions (excluding gravity) from a unified point of view. In contrast to this assumption, nonlinear spinor theory is guided by the idea that in a first step the pointlike particles should be calculated in a selfconsistent manner, while in a second step the masses of the bosons and the coupling constants have to be derived. In the third and last step the formation of arbitrary complicated clusters of these "elementary" fermions and bosons and their mutual interactions are then considered.

The idea of the reversed role of hadrons and leptons was first proposed by Saller [7] who treated a lepton model of the nonlinear spinor field. However, Saller gave no qualitative or quantitative hint with respect to quark dynamics, quark confinement, etc., but only mentioned a possible identification of dipole ghosts and quarks. This lepton model was not pursued further. Rather Dürr [3] and Saller [8] tried to convert the nonlinear spinor field into a renormalizable selfinteracting gauge field, from which the indefinite metric can be gauged away or eliminated by subsidiary conditions. Later on, the quark idea was completely doubted, Dürr and Saller [9].

Hence, if we want to establish a lepton-hadron model of the nonlinear spinor field, a quantitative formulation of the corresponding dynamics is needed. In this preliminary paper for simplicity we only consider a lepton-hadron system which consists of the electron e^- and the corresponding d quark. In this case, the original version of the nonlinear spinor field theory can directly be used for the formulation of a lepton-quark dynamics. According to Heisenberg [2] the regularization of the spinor field can be completely achieved by employing a regularized spinor field propagator. If the spinor field is denoted by $\psi(x)$, the simplest expression for the corresponding regularized propagator reads, cf. Stumpf [1]

$$F(x-x') := \langle 0 | T \psi(x) \bar{\psi}(x') | 0 \rangle = (2\pi)^{-4} \int \frac{(-\bar{p}+\mu)^2(-\bar{p}+m)}{(p^2-\mu^2)^2(p^2-m^2)} \cdot e^{i p (x-x')} \, d^4 p \,. \tag{1}$$

It can be shown, cf. Stumpf [1], that F just contains one physical particle, one monopole ghost and one dipole ghost. The quantum states, eigenvalues and other global physical observables can be derived from the state functionals

$$|\mathfrak{T}(j,a)\rangle := \sum_{n=1}^{\infty} \int \langle 0 | T \Psi(x_1) \ldots \Psi(x_n) | a \rangle \cdot | D(x_1 \ldots x_n) \rangle \, d^4 x_1 \ldots d^4 x_n \tag{2}$$

resp.

$$|\mathfrak{F}(j,a)\rangle := [\exp - \tfrac{1}{2} \int j(x) \hat{F}(x-x') \cdot j(x') \, d^4 x \, d^4 x'] \, |\mathfrak{T}(j,a)\rangle \tag{3}$$

where $\Psi(x)$ is the corresponding Hermitean spinor to $\psi(x)$. and $\hat{F}$ the corresponding Hermitean propagator.

The calculation of these functionals and other operations can be performed by means of the functional quantum theory. In particular, effective equations for the quantities

$$\varphi(x_1 \ldots x_n \mid a) := \langle D(x_1 \ldots x_n) \mid \mathfrak{F}(j, a)\rangle \quad (4)$$

are provided by this formalism. For the one-particle sector we have $\varphi(x \mid a) \equiv \langle 0 \mid \Psi(x) \mid a\rangle$ and this matrix element must satisfy a corresponding equation

$$K \varphi(x \mid a) = 0 \quad (5)$$

where K is the selfenergy operator of the one-particle sector. Decomposing the Hermitean spinor operator $\Psi(x)$ into $\psi(x)$ and $\bar{\psi}(x)$ the corresponding matrix elements $\langle 0 \mid \psi(x) \mid a\rangle$ and $\langle 0 \mid \bar{\psi}(x) \mid a\rangle$ must satisfy eigenvalue equations

$$K_1 \langle 0 \mid \psi(x) \mid a\rangle = 0; \quad \langle 0 \mid \bar{\psi}(x) \mid a\rangle K_1 = 0 \quad (6)$$

which can be derived from (5). By a spectral decomposition of $F(x-x')$, the propagator can be directly related to the matrix elements $\langle 0 \mid \psi(x) \mid a\rangle$ for the various one-particle states $\mid a\rangle$, i.e. the properties of F determine the properties of the matrix elements $\langle 0 \mid \psi(x) \mid a\rangle$ and vice versa. If $F(x-x')$ is given by (1) it can be shown, cf. Stumpf [1], that all $\langle 0 \mid \psi(x) \mid a\rangle$ must satisfy the equation

$$\begin{aligned} K_1 \langle 0 \mid \psi(x) \mid a\rangle \equiv (-i\gamma^\mu \partial_\mu + m) & \\ \cdot (-i\gamma^\nu \partial_\nu + \mu)^2 & \\ \cdot \langle 0 \mid \psi(x) \mid a\rangle = 0 . & \end{aligned} \quad (7)$$

Provided that this selfconsistency condition is satisfied by the spinor field, from this equation it follows that the field operator in the one-particle sector describes a mixture of a real particle and ghost particles. Hence if we identify the real particle with a lepton and the dipole ghost with a quark, the effective equations for the matrix elements

$$\varphi(x_1 \ldots x_n \mid a) = \langle 0 \mid : \Psi(x_1) \ldots \Psi(x_n) : \mid a\rangle$$

cannot properly describe a lepton-hadron dynamics as they are referred to the dynamics of an "unobservable" mixture of both kinds of particles. Hence a proper lepton-hadron dynamics can only be established if we succeed to separate these both constituent parts of the fundamental field.

In order to do this, we observe that a general solution of Eq. (7) can be written in the following form

$$\begin{aligned} \langle 0 \mid \psi(x) \mid a\rangle = & \int F_q(x-x')\, l(x')\, \mathrm{d}^4x' \\ & + \int F_l(x-x')\, q(x')\, \mathrm{d}^4x' \end{aligned} \quad (8)$$

where the quantities F_q, q, F_1, l satisfy the equations

$$\begin{aligned} (-i\gamma^\mu \partial_\mu + m) F_l(x-x') &= \delta(x-x'); \\ (-i\gamma^\mu \partial_\mu + m)\, l(x) &= 0, \\ (-i\gamma^\nu \partial_\nu + \mu)^2 F_q(x-x') &= \delta(x-x'); \\ (-i\gamma^\nu \partial_\nu + \mu)^2 q(x) &= 0, \end{aligned} \quad (9)$$

i.e. $l(x) \equiv \langle 0 \mid l(x) \mid a\rangle$ describes a physical particle of mass m, the lepton, while $q(x) \equiv \langle 0 \mid q(x) \mid a\rangle$ describes a mixture of a monopole and a dipole ghost and can be identified with the quark. If we consider the corresponding linear theory, then for $\mid a\rangle \equiv$ lepton state, the matrix element $\langle 0 \mid q(x) \mid a\rangle$ vanishes, i.e. we have

$$\begin{aligned} & \langle 0 \mid \psi(x) \mid \text{lepton}\rangle \\ & = \int F_q(x-x') \langle 0 \mid l(x') \mid \text{lepton}\rangle \end{aligned} \quad (10)$$

while for $\mid a\rangle \equiv$ quark state, the matrix element $\langle 0 \mid l(x) \mid a\rangle$ vanishes, i.e. we have

$$\begin{aligned} & \langle 0 \mid \psi(x) \mid \text{quark}\rangle \\ & = \int F_l(x-x') \langle 0 \mid q(x') \mid \text{quark}\rangle . \end{aligned} \quad (11)$$

Similar formulae hold for many-particle quark and lepton states of the linear theory.

It is reasonable to transfer these relations by definition to the nonlinear theory in order to distinguish lepton and quark matrix elements. In this case, for a mixed state of n leptons and m quarks, we put with $\varrho = n + m$

$$\begin{aligned} \varphi(x_1 \ldots x_\varrho \mid a) = \int & \hat{F}_q(x_1 - z_1) \ldots \hat{F}_q(x_n - z_n) \\ & \cdot \hat{F}_l(x_{n+1} - y_1) \ldots \hat{F}_l(x_\varrho - y_m) \\ & \cdot X_{n,m}(z_1 \ldots z_n, y_1 \ldots y_m) \\ & \cdot \mathrm{d}^4z_1 \ldots \mathrm{d}^4z_n\, \mathrm{d}^4y_1 \ldots \mathrm{d}^4y_m \end{aligned} \quad (12)$$

where $\hat{F}$ is the corresponding Hermitean propagator to F. By means of this formula the effective equations for the unobservable ψ-amplitudes can be rewritten into equations for the amplitudes of the observable lepton field and the confined quark fields. As the kernels of the effective equations are calculated by the unobservable but regularized ψ-field, no divergencies occur and this property will also be shared by the new equations for the lepton-hadron systems.

In a preliminary test calculation one can do without the selfconsistent calculation of the one-particle sector, i.e. one can assume the pointlike fermion particles (and ghosts) to be known. Then only the bosons and the higher aggregates and their reactions have to be calculated. Performing this program the boson functionals have to start with $l \otimes q$, $l \otimes l$ or $q \otimes q$ representations, while, for instance, the observable hadron functionals must start with a $q \otimes q \otimes q$ representation. In functional quantum theory of the nonlinear spinor field, boson state functionals have been studied in the old version since a long time. These calculations can be easily transferred into the new version. Concerning hadron functional states, corresponding functional Fadeev equations have been derived in the framework of functional quantum theory, Ramacher [10], which allow the calculation of hadron masses from quark states in a completely relativistic invariant way with inclusion of confinement. Performing such three-quark calculations, it has to be emphasized that the resulting bound states are not on the mass shell of the single quarks involved, so that such a bound state is not forced to belong to the unphysical part of the state space.

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